# A MICROSTRIP PLANAR DISK 3dB QUADRATURE HYBRID

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## ABSTRACT

**A 3dB quadrature hybrid in the form of a four-port microstrip planar disk circuit is presented. Experimental results verifying the junction design are given.**

## INTRODUCTION

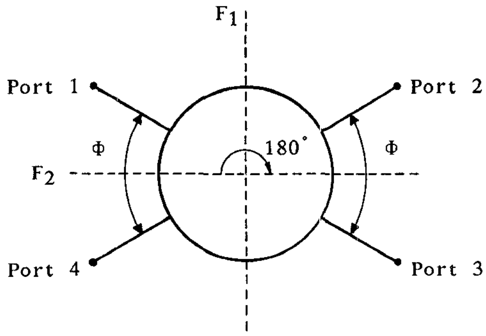
Planar disk circuit devices have been shown to have a number of advantages over junctions based on transmission line designs, notably that topologically simple devices of reasonable dimensions can be designed for high frequency operation using planar disk circuits.

The 3dB rat-race hybrid for example has ports located at $\Phi=0^\circ$, $90^\circ$, $135^\circ$ and $225^\circ$ around the periphery of the disk [1]. The circuit operates by utilising the field distribution of the dipole mode resonance of the microstrip disk. The operation of the device, therefore, depends essentially on a single mode only.

For a quadrature hybrid, normal mode analysis indicates that the symmetry shown in Fig. 1 must be present, i.e. $180^\circ$ rotational symmetry and reflection symmetry about the $F_1$ and $F_2$ planes.

Since the device is symmetrical, then it is straight forward to show that for a loss free circuit, if the device is matched, two pairs of ports are isolated. Furthermore, the phase difference between the coupled waves is $90^\circ$. This then forms the basis for the design of the 3dB quadrature hybrid. It now only remains to choose the necessary port positions for which the circuit is matched and equal power balance is obtained.

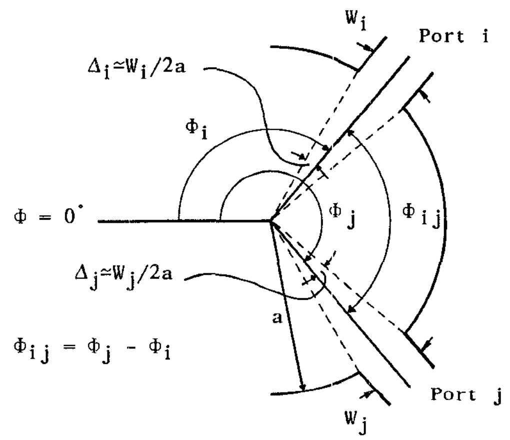
Fig. 1 The Symmetry of the Quadrature Hybrid Junction




## METHOD OF ANALYSIS

The impedance matrix elements of the multiport disk circuit of Fig. 2 are evaluated from the Greens Function using the relationship of eq.(1) [2].

Fig. 2 Parameters Used in the Analysis of a Multiport Disk Circuit



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$$Z_{ij} = \frac{1}{W_i \; W_j} \int_{W_i} \int_{W_j} G(\, s \mid s_o \,) \; ds \; ds_o \qquad (1)$$

In this expression $W_i$ and $W_j$ represent the widths of the $i^{th}$ and $j^{th}$ ports respectively.

The Greens Function for a circular, edge magnetic wall, microstrip disk resonator [2] is

$$G(\, r,\Phi \mid r_o,\Phi_o \,) = \frac{j\omega\mu d}{\pi} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{\sigma_m \; J_m(k_{mn}r)}{(k_{mn}^2 - k^2)} \cdot \frac{J_m(k_{mn}r_o) \; \cos m(\Phi - \Phi_o)}{(a^2 - m^2/k_{mn}^2) \; J_m^2(k_{mn}a)} \right\} \qquad (2)$$

where

$$\sigma_m = \begin{cases} 1 \;, & m = 0 \\ 2 \;, & m \neq 0 \end{cases} \qquad (3)$$

The dielectric height, the permeability and the dielectric constant of the substrate are represented by d, $\mu$ and $\epsilon r$ respectively. In the expression of eq.(2) the static capacitance component of the Greens Function is not included although the effect of the static capacitance is included when generating theoretical frequency responses. The values of $k_{mn}$ must satisfy the following boundary condition.

$$\left. \frac{\partial \; J_m(k_{mn}r)}{\partial r} \right|_{r=a} = 0 \qquad (4)$$

Evaluating eq.(1) in conjunction with the Greens Function of eq.(2) yields the following expressions for the impedance matrix elements. The diagonal impedance matrix elements, $Z_{ii}$, all take the same value and are written as

$$Z_{ii} = \frac{j\omega\mu d}{\pi} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{\sigma_m}{(k_{mn}^2 - k^2) \; (a^2 - m^2/k_{mn}^2)} \cdot \left[ \frac{\sin(\, mW_i/2a \,)}{mW_i/2a} \right]^2 \right\} \qquad (5)$$

The other off-diagonal impedance matrix elements, $Z_{ij}$, are written as

$$Z_{ij} = \frac{j\omega\mu d}{\pi} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{\sigma_m \; \cos m(\Phi_i - \Phi_j)}{(k_{mn}^2 - k^2) \; (a^2 - m^2/k_{mn}^2)} \cdot \left[ \frac{\sin(\, mW_i/2a \,)}{mW_i/2a} \; \frac{\sin(\, mW_j/2a \,)}{mW_j/2a} \right] \right\} \qquad (6)$$

where $\Phi_i$ and $\Phi_j$ are the angular locations of the $i^{th}$ and $j^{th}$ ports respectively.

In evaluating the impedance matrix the effective radius of the disk is used, as is the dynamic dielectric constant [3]. The frequency variation of the characteristic impedance, Zo, and the effective widths of the microstrip lines are also taken into account [2].

Since the symmetry of Fig. 1 has been imposed on the junction, the impedance matrix elements can be written down in terms of the eigenimpedances.

$$\begin{aligned} Z_{11} &= (\, Z_1 + Z_2 + Z_3 + Z_4 \,) \,/\, 4 \\ Z_{12} &= (\, Z_1 - Z_2 + Z_3 - Z_4 \,) \,/\, 4 \\ Z_{13} &= (\, Z_1 + Z_2 - Z_3 - Z_4 \,) \,/\, 4 \\ Z_{14} &= (\, Z_1 - Z_2 - Z_3 + Z_4 \,) \,/\, 4 \end{aligned} \qquad (7)$$

This allows the eigenimpedances to be expressed in terms of the impedance matrix elements as follows

$$\begin{aligned} Z_1 &= Z_{11} + Z_{12} + Z_{13} + Z_{14} \\ Z_2 &= Z_{11} - Z_{12} + Z_{13} - Z_{14} \\ Z_3 &= Z_{11} + Z_{12} - Z_{13} - Z_{14} \\ Z_4 &= Z_{11} - Z_{12} - Z_{13} + Z_{14} \end{aligned} \qquad (8)$$

It should be noted that, although not necessary, the impedance matrix elements have been normalised to $Z_o$.

The eigenreflections are related to the eigenimpedances via the relationship of eq.(9).

$$S_i = \frac{Z_i - 1}{Z_i + 1} \qquad (9)$$

Expressing the scattering matrix elements in terms of the eigenreflections allows the S-matrix to be computed directly.

$$S_{11} = ( S_1 + S_2 + S_3 + S_4 ) / 4$$

$$S_{12} = ( S_1 - S_2 + S_3 - S_4 ) / 4$$

$$S_{13} = ( S_1 + S_2 - S_3 - S_4 ) / 4 \qquad (10)$$

$$S_{14} = ( S_1 - S_2 - S_3 + S_4 ) / 4$$

## EXPERIMENTAL RESULTS

Examination of the frequency response of the symmetrical device of Fig. 1 as $\Phi$ varies from $0^\circ$ through to $90^\circ$ showed the device to exhibit match, isolation and equal power division when the angle between ports 1 and 4 and ports 2 and 3 was chosen to be $65^\circ$ (i.e. when $\Phi=65^\circ$). The theoretical response of the device for $\Phi=65^\circ$ is shown in Fig. 3 for the frequency range 6 – 8 GHz. The results are generated for a substrate having a relative permittivity of 2.5 and a height of 1.52 mm. The physical radius of the disk is 15 mm and port impedances of 50 Ω are used. Fig. 4 plots $\theta_{13} - \theta_{14}$ vs frequency thus indicating the quadrature nature of the junction. The quantity $\theta_{13} - \theta_{14}$ represents the phase difference between the waves emerging from the coupled ports.

Fig. 3 The Theoretical Response of the Microstrip Planar Disk 3dB Quadrature Hybrid

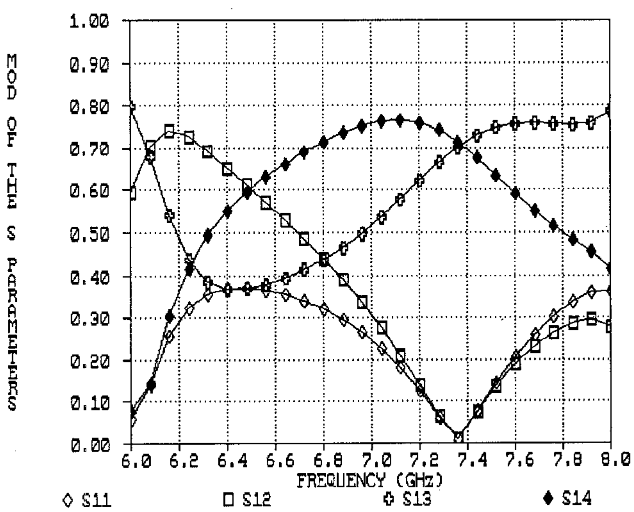
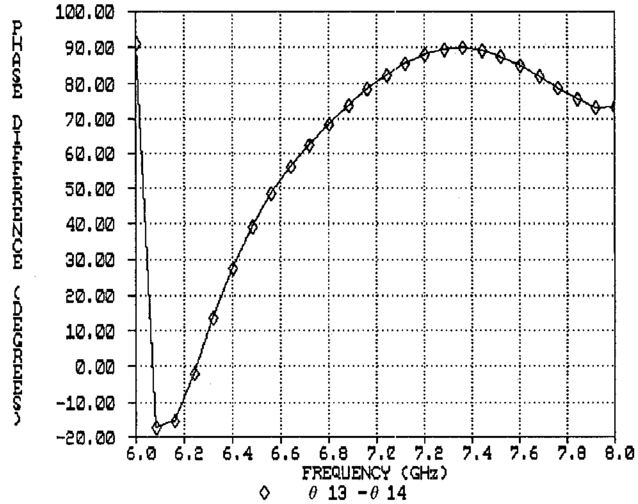
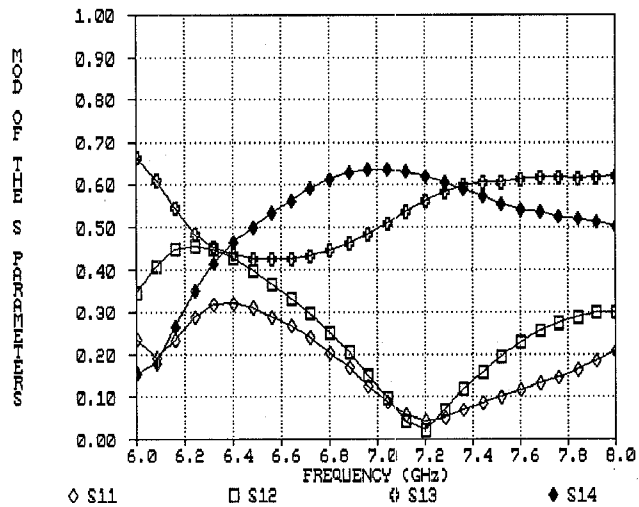



Fig. 4 The Theoretical Quadrature Phase Difference Between Coupled Ports




The experimental frequency response is shown in Fig. 5. This shows good agreement with the theoretical response of Fig. 3.

Fig. 5 The Experimental Respone of the Microstrip Planar Disk 3dB Quadrature Hybrid




If one accepts an $S_{11} < 0.1$ as a tolerable match condition, the isolation is always below 0.2 and this results in a usable bandwidth of 6.8% experimentally and 3% theoretically; the higher experimental bandwidth being directly attributable to losses.

## CONCLUSIONS

A 3dB quadrature hybrid has been successfully designed in planar disk form. Using this simple multiport planar disk approach, a device has been produced which operates at 7.2 GHz and exhibits a bandwidth of 6.8%. It is now relatively straight forward to design such a device at even higher frequencies by simply reducing the radius of the disk. As an example, if the disk radius is 12 mm the device operates at 9.1 GHz. The dimensional tolerance required is not too demanding for fabrication. The device should be of particular use in microwave modulators, where conventionally the 3dB quadrature transmission line hybrid has been used.

## ACKNOWLEDGEMENTS

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